MODULE 5: TIME VALUE OF MONEY / CAPITAL INVESTMENTS
Chapter 11

Learning Objectives:		Topic*	Ch & Time
5.1	Understand the relationship between risk and return.	F	Ch 11 8 hours
5.2	Determine present value and future value cash flows.	F	
		Module 5 Total Hours = 8	

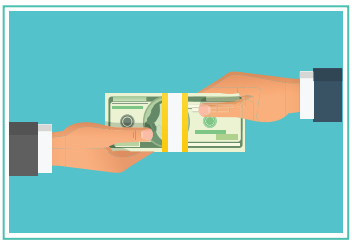
NOTES:

LEARNING OBJECTIVE 5.1:

Understand the relationship between risk and return.

Time Value of Money is a tool that is used to help companies make decisions to invest in resources and to finance those resources. This is a stand alone module because it is used for financing decisions (e.g. to borrow money) and for investing decisions (e.g. to acquire a piece of equipment).

Return *of* vs. Return *on* Investment:
When we invest, we expect to get more than we invest!



Invest $1,000 it becomes $1,050

$1,000 Return OF

$50 Return ON

Factors to consider when assessing "Return on" investment

1. Time - Would you prefer a $50 return in one year or a $60 return in two years?
2. Common Size Measurement - Rate of Return is needed to compare alternative investments of different amounts.

Recommended Video:
http://www.investopedia.com/video/play/return on investment basics/

Which is better $1,000 or $10,000 return on investment? (Rate of Return – assumes one year)

$$\frac{\$ \text{ Amount of Return on Investment}}{\$ \text{ Amount of Investment}}$$

The $1,000 return is for a $10,000 investment or the $10,000 return is for a $1,000,000 investment. What is your Rate of Return on these two investments?

Can you have a negative rate of return?

Uncertainty Creates Risk

In the absence of clairvoyance, we forecast possible future outcomes. The chance of an undesirable outcome occurring creates risk. The Expected Rate of Return takes into consideration the chances of an outcome occurring. It is a weighted average of possible rates of return.

EXPECTED RATE OF RETURN EXAMPLE

INVESTMENT A: $100,000

Probability of Occurring	.2	.4	.4
Possible Return	$10,000	$5,000	$2,000

Expected Return = (.2)($10,000) + (.4)($5,000)+(.4)($2,000)=$4,800

Expected Rate of Return (ERR) = $4,800/$100,000 = 4.8%
Or
Expected Rate of Return=
(.2)(10%) + (.4)(5%) + (.4)(2%) = 4.8%

$10,000/$100,000= .1 or 10%

$5,000/$100,000=.05 or 5%
$2,000/$100,000=.02 or 2%

INVESTMENT B: $100,000

Probability of Occurring	.2	.5	.3
Possible Return	$30,000	$5,000	<$4,000>

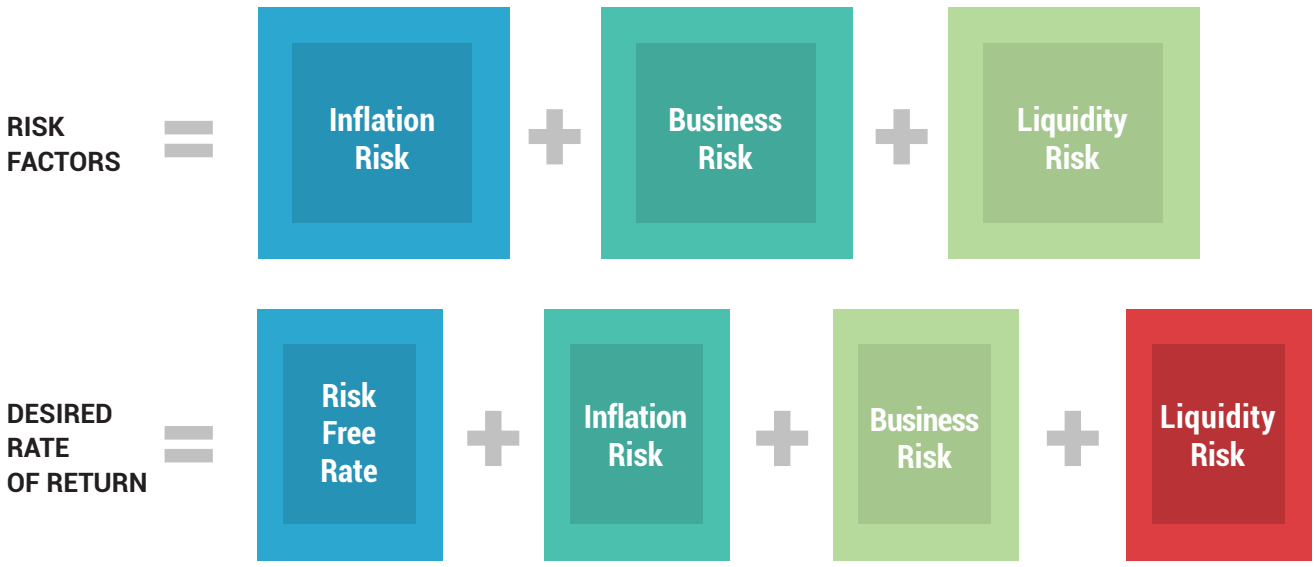
Expected Return = (.2)($30,000) + (.5)($5,000) +(.3)(<$4,000>) = $7,300

Expected Rate of Return (ERR) = $7,300 / $100,000 = .073 or 7.3%
OR
Expected Rate of Return=
(.2)(30%) + (.5)(5%) + (.3)(<4%>) = 7.3%

$30,000/$100,000 = .3 or 30%

$5,000/$100,000= .05 or 5%
<$4,000>/$100,000=<.04> or <4%>

Would you invest in investment A or B? Why?



Personal Risk Preferences: Risk Taker vs. Risk Averse

$ Recommended Video: "Along Came Polly" (extreme example of risk taker vs. a risk averse)

LEARNING OBJECTIVE 5.2:

Determine present value and future value cash flows.

Simple vs. Compound Interest

Principal x Rate x Time = Interest

(The interest rate used takes into consideration the risk factors above.)

Simple Interest - Interest computed only on the amount borrowed (principal).

Assume $100,000 is invested on 1/1/16 for three years

12/31/2016	$100,000 x .1 x 1 =	$10,000
12/31/2017	$100,000 x .1 x 1 =	$10,000
12/31/2018	$100,000 x .1 x 1 =	$10,000
	Total Interest	$30,000
	Return of Principal	$100,000

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Compound Interest: Interest added to principal at specified points in time.

Compounding Period: Point where interest is added to Principal.

Annual Compounding: Interest added to principal at end of each year before calculating interest for the next year.

Assume $100,000 is invested on 1/1/16 for three years

12/31/2016	$100,000 x .1 x 1 =	$10,000
12/31/2017	$110,000 x .1 x 1 =	$11,000
12/31/2018	$121,000 x .1 x 1 =	$12,100
	Total Interest	$33,100
	Return of Principal	$100,000

Where did the extra $3,100 come from?

What if $100,000 invested 1/1/16 was Compounded Semi Annually?

6/30/2016	$100,000 x .1 x 1/2 =	$5,000.00
12/31/2016	$105,000 x .1 x 1/2 =	$5,250.00
6/30/2017	$110,250 x .1 x 1/2 =	$5,512.50
12/31/2017	$115,762.50 x .1 x 1/2 =	$5,788.13
6/30/2018	$121,550.63 x .1 x 1/2 =	$6,077.53
12/31/2018	$127,628.16 x .1 x 1/2 =	$6,381.41
	Total Interest	$34,009.56
	Return of Principal	$100,000

Why has the return **ON** investment increased here?

What if compounded monthly for 3 years? Using the previous process this will require a large number of computations.

1. How many compoundings?
2. How will the amount of interest differ from above?

The desire to not do a large number of compoundings led to: $(1 + r/c)^n$

Where r = Annual Rate
c = Number of compoundings per year
n = Total number of compoundings

Using semiannual compounding from above for three years:
$(1 + .10/2)^6 = 1.340095$ What does this mean?

Recommended Video: https://www.bettermoneyhabits.com/saving budgeting/ saving forfuture/time value money.html

FUTURE VALUE OF AMOUNT OF $1: $FV = PV \times (1 + r/c)^n$

$100,000 invested 3 years compounded semiannually will become?
Use calculator to solve:
$FV = \$100,000 \times (1 + .10/2)^6 = \$134,009.50$

Compare formula result 1.340095 to Table 1 in Chapter 11.
What does the 1.340095 represent?

What would $100,000 invested 3 years compounded monthly become?
$FV = \$100,000 \times (1 + .10/12)^{36} = \$134,818.20$

PV = Lump sum of money today

FV = Lump sum of money at some point in the future

r = Annual Rate of Return

c = Number of compoundings in a year

n = Total number of compounding over time period

PV = $100,000	r = 10%	c = 12	n = 36	FV = ?

PRESENT VALUE OF $1: $PV = FV \times \frac{1}{(1 + r/c)^n}$

You want to have $134,009.50 in 3 years. How much would you have to invest today to achieve this goal assuming you can earn 10% compounded semiannually?

$FV = PV \times (1 + r/c)^n$

$\$134,009.50 = PV \times (1 + .10/2)^6$

$FV \times \frac{1}{(1 + r/c)^n} = PV$ $\qquad$ $\$134,009.50 \times \frac{1}{(1 + .10/2)^6} = PV$

Another View: $100,000 today is the equivalent of $134,009.50 3 years from today ASSUMING the interest rate is 10% and is compounded semiannually.

Compare formula result (.746216) to Table 2 in Chapter 11.
What does .746216 represent?

Assume you want to have $100,000 in 5 years how much must you invest today if you can earn 8% that is compounded quarterly?

$PV = FV \times \frac{1}{(1 + r/c)^n}$

$PV = 100{,}000 \times \frac{1}{(1 + 8/2)^{20}} = \$67{,}297.10$

PV = Lump sum of money today

FV = Lump sum of money at some point in the future

r = Annual Rate of Return

c = Number of compoundings in a year

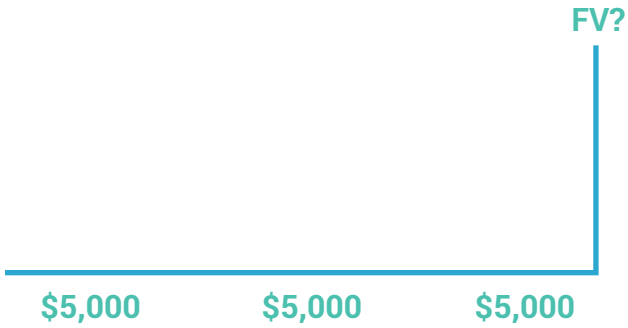
n = Total number of compounding over time period

FV = $100,000	r = 8%	c = 4	n = 20	PV =

Annuities - *Equal cash payments* at *equal time intervals* with the *same interest rate* over the entire time period. Interest is compounded at each payment date.

FUTURE VALUE OF AN ANNUITY: Amount that payments and interest accumulates to at a specified date in the future.

3 annual payments of $5,000 starting 1/1/16. How much will you have when the last payment is made if you can earn 10% interest?



Date	Payment	Interest	Future Value
1/1/2016	$5,000	$0	$5,000
1/1/2017	$5,000	$500	$10,500
1/1/2018	$5,000	$1,050	$16,550

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Math formula $FV = Annuity \times \frac{[(1 + r/c)^n - 1]}{r/c}$

PMT = Annuity

c = Number of Payments / Compoundings per year

r = Annual rate of interest

n = Number of payments

FV = Future Value

FV = ?	PMT = $5,000	r = 10%	c = 1	N = 3

Look at Table 3 in Chapter 11. Across the top to 10%, down left side to 3 payments. What does this number at the intersection of the two numbers mean?

What IF 15 annual payments were made with a 10% annual interest?

FV = ?	PMT = $1,000	r = 10%	c = 1	n = 15

FV = $31,772.48

NOTE THAT NUMBER COMES OUT NEGATIVE!

WHAT IF you wanted to have $31,772.48 after making 15 payments how much would each payment have to be?

FV= $31,772.48	PMT= **$1,000**	c= 1	n= 15	r=10%

WHAT IF you wanted to have $1,000,000 after making monthly payments for 40 years and you could earn an 8% annual interest rate? How much must the monthly payments be?

FV= $1,000,000	PMT= **$3,860.16**	c= 12	n=480	r=8%

HOW MUCH would you have if you made monthly payments of $300 per month for 40 years and could earn 5%?, 6%?, 10%?

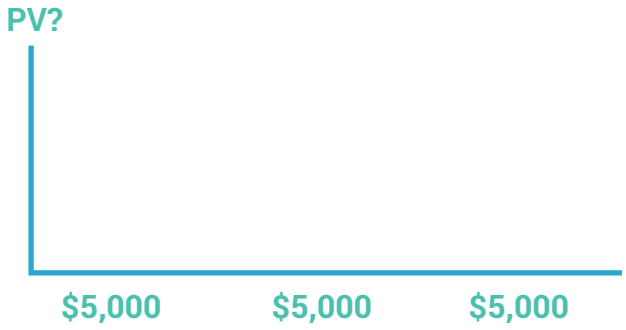
FV= **$457,806.05**	PMT=$300	r=5	c = 12	n=480
FV= **$597,447.22**	PMT=$300	r=6	c = 12	n=480
FV= **$1,897,223.87**	PMT=$300	r=10	c = 12	n=480

Note: you only have to change the rate after first calculation is made.

PRESENT VALUE OF AN ANNUITY: What is an annuity worth today?

Formula: $$PV = \text{Annuity} \times \frac{[1-1/(1+r/c)^{n}]}{r/c}$$

What are three annual $5,000 payments worth today if the interest rate is 10% and the first payment is one year from today?



PV = **$12,434.26**	PMT= $5,000	c= 1	n=3	r=10%

Look at Table 4 in Chapter 11. Across the top to 10% down left side to 3 payments. What does this number at the intersection of the two numbers mean?

Recommended Website:
http://www.getobjects.com/Components/Finance/TVM/diagram.html

Making it more intuitive:

How much could you borrow if you agreed to pay $5,000/ year for 3 years at 10% annual interest rate?

OR

How much would you have to invest today to be able to withdraw $5,000 a year for three years with the first $5,000 withdrawal one year from today?

Date	Payment	Interest	Principal	Present Value
1/1/2016				$12,434.26
1/1/2017	$5,000	$1,243.43	$3,756.57	$8,677.87
1/1/2018	$5,000	$867.77	$4,132.23	$4,545.64
1/1/2019	$5,000	$454.56	$4,545.64	$0

When you retire at age 55 you want to be paid $100,000 a year for 15 years. How much must you invest at age 54 so you can withdraw $100,000 a year starting at age 55 for 15 years if you can earn a 5% interest rate?

PV=**$1,037,965.80**	PMT=$100,000	c= 1	n= 15	r=5%

What if you could get 7%?

PV=**$910,791.40**	PMT=$100,000	c= 1	n= 15	r=7%

You just borrowed $20,000 to buy a car. The interest rate is 8% and the payments are monthly for 3 years. How much are your payments?

PV= $20,000	PMT= **$626.73**	r= 8%	c= 12	n= 36

What your payments be if you could get 5% interest?

PV= $20,000	PMT= **$599.42**	r= 5%	c= 12	n= 36

$ With your group create your own problem and prepare to report.

How do you know which method to use?

You want to have $40,000 in 5 years, how much do you need to invest annually starting today to achieve this goal if the interest rate is 6%?

Is this a
- FV of $1?
- PV of $1?
- FV of Annuity?
- PV of Annuity?

Use the following flowchart: